Submitted by-Aditya Gautam

Roll No-12

MSc 3rd semester

Date of Experiment-01/10/2020

Date of Submission-05/10/2020

**Experiment No-01**

**Topic-** DRAWINGOF A RANDOM SAMPLE AND ESTIMATION OF PARAMETERS OF A BIVARIATE NORMAL DISTRIBUTION

**Problem-** Draw a random sample of size 10 from a bivariate normal population having the mean vector and covariance matrix as

 

Hence, estimate the parameters based on the random sample thus obtained (assuming that the parameters are unknown).

**Theory-**

The procedure of drawing a random sample from the bivariate normal distribution say

 is as follows:

**Step 1-** We write down the marginal distribution of X, which is 

**Step 2-** We draw a random number from uniform(0,1) distribution and we denote it by R

**Step 3**- We consider the CDF of X given by

 and we set



Where, is the quantile function of X. The value of x thus obtained is a random value of X.

**Step 4-** We write down the conditional distribution of Y/X=x which is given by



Where, 

And x is the value of X obtained in step 3.

**Step 5**- Since the conditional distribution of Y/X is a given univariate normal, we repeat the step 2 and 3 in order to obtain a random value of Y/X=x.



**Step 6-** Step 1-5 are repeated n times in case a random sample of size n from (X,Y) is desired. The ‘n’ pairs of values  thus obtained constitute random sample of size n from the bivariate normal population X,Y . The maximum likelihood estimates of the parameters ,, ,, are calculated from the sample as shown below-



**Calculation-**

The R-program for obtaining the random sample and estimating the parameters from the given BND is as follows-

mx=12; my=15; sdx=8; sdy=5

rho=40/(sdx\*sdy)

rho

r<-runif(10,0,1)

x<-mat.or.vec(10,1)

for(i in 1:10){

x[i]<-qnorm(r[i],mx,sdx)}

x

my\_x<-my+((rho\*sdy/sdx)\*(x-mx))

sdyx<-sqrt(sdy^2\*(1-(rho^2)))

r1<-runif(10,0,1)

y<-mat.or.vec(10,1)

for(i in 1:10){

y[i]<-qnorm(r1[i],my\_x[i],sdyx)}

y

mxhat<-mean(x)

mxhat

myhat<-mean(y)

myhat

sx\_hat<-sqrt(var(x))

sx\_hat

sy\_hat<-sqrt(var(y))

sy\_hat

rho\_hat<-cov(x,y)/(sx\_hat\*sy\_hat)

rho\_hat

**Conclusion-**

The random sample of size 10 from the BND with given parameters  and is given by-

(1.673798, 8.546124), (7.312850, 12.070531), (14.457159, 16.53572), (21.502800, 20.939250),

(14.241883, 16.401177), (2.439168, 9.024480), (14.134589, 16.334118), (32.028724, 27.517953),

(24.277234, 22.673272), (10.261240, 13.913275)

The value of the estimated parameters are-

